Model Answer
Physics – Paper II: Electrostatics and Magnetostatics
Paper Code: AS-2768

1. (i) A field is irrotational if
(a) grad $\mathbf{A} = 0$ (b) div $\mathbf{A} = 0$ (c) Curl $\mathbf{A} = 0$ (d) None of these

Ans. : (c)

(ii) The electric charge A and B are attracted to each other. Electric charge B and C repel each other. If A and C are held close together they will
(a) attract (b) repel
(c) not affect each other (d) more information needed

Ans. : (a)

(iii) In a conservative field of electrostatics,
(a) Curl $\mathbf{E} = 0$ (b) Div $\mathbf{E}=0$ (c) Curl $\mathbf{E} \neq 0$ (d). none of these

Ans. : (a)

(iv) The potential energy of a electric dipole in electric field
(a) maximum when it is aligned with the field
(b) maximum when it is anti-parallel to the filed
(c) maximum when it is perpendicular to the filed
(d) none of the above

Ans. : (b)

(v) A molecule in which the center of positive charge does not coincides with center of negative charge is called
(a) Polar molecule (b) non polar molecule (c) water molecule (d) none

Ans. : (a)

(vi) Equation of continuity is
(a) $\nabla . J - \partial\rho / \partial t = 0$ (b) $\nabla . J + \partial\rho / \partial t = 0$
(c) $\nabla \rho - \partial J / \partial t = 0$ (d) $\nabla \rho + \partial J / \partial t = 0$

Ans. : (b)

(vii) In the absence of an external electric field on a dipolar substance, the electric dipoles are:
(a) parallel (b) alternatively anti-parallel
(c) randomly oriented (d) none

Ans. : (c)

(viii) Electrostatic field inside a perfect conductor is
(a) same everywhere (b) infinite
(c) zero (d) none of these

Ans. : (c)

(ix) Dipole moment per unit volume of the material is called

(a) polarization (b) polarizability

(c) susceptibility (d) none of these

Ans. : (a)

(x) Ampere's circuital law can be applied __________ the conductor.

(a) Inside (b) Outside

(c) Both (a) and (b) (d) None of these

Ans. : (c)

Section – II **(5 × 4 =20)**

Note: *Answer any* **five** *questions from this section. All questions carry equal marks.*

2. (a) Define scalar product of two vectors and derive an expression for the angle between them.

Ans. If two quantities are represented by vector ***A*** and ***B*****,** then the scalar product of two vectors ***A*** and ***B*** is written as

$$\boldsymbol{A}\cdot\boldsymbol{B}=|\boldsymbol{A}||\boldsymbol{B}|\cos\theta$$

where, θ is angle between two vectors and

$$\cos\theta=\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}||\boldsymbol{B}|}\Rightarrow\theta=\cos^{-1}\left(\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{|\boldsymbol{A}||\boldsymbol{B}|}\right)$$

If $\boldsymbol{A}=\boldsymbol{a_x}\hat{\boldsymbol{i}}+\boldsymbol{a_y}\hat{\boldsymbol{j}}+\boldsymbol{a_z}\hat{\boldsymbol{k}}$ and $\boldsymbol{B}=\boldsymbol{b_x}\hat{\boldsymbol{i}}+\boldsymbol{b_y}\hat{\boldsymbol{j}}+\boldsymbol{b_z}\hat{\boldsymbol{k}}$

then

$$\boldsymbol{A}\cdot\boldsymbol{B}=\boldsymbol{a_x b_x}+\boldsymbol{a_y b_y}+\boldsymbol{a_z b_z}$$

$$\cos\theta=\frac{\boldsymbol{a_x b_x}+\boldsymbol{a_y b_y}+\boldsymbol{a_z b_z}}{\sqrt{\boldsymbol{a_x^2}+\boldsymbol{a_y^2}+\boldsymbol{a_z^2}}\sqrt{\boldsymbol{b_x^2}+\boldsymbol{b_y^2}+\boldsymbol{b_z^2}}}$$

(b) The resultant of two vectors P and Q is a vector R. After reversing the direction of Q their resultant is D. Prove that $(R^2+D^2) = 2(P^2+Q^2)$

Ans. The resultant of two vector **P** and **Q** is given by

$$\mathbf{R}^2 = \mathbf{P}^2+\mathbf{Q}^2+2\mathbf{PQ}\cos\theta \qquad (1)$$

After reversing the direction of **Q** the new vector is **-Q**

The resultant is given by

$$\mathbf{D}^2 = \mathbf{P}^2+\mathbf{Q}^2 - 2\mathbf{PQ}\cos(180° - \theta) = \mathbf{P}^2+\mathbf{Q}^2 - 2\mathbf{PQ}\cos\theta \qquad (2)$$

Then, from eqn(1) + eqn(2)

$$\mathbf{R}^2+\mathbf{D}^2= \mathbf{P}^2+\mathbf{Q}^2+2\mathbf{PQ}\cos\theta + \mathbf{P}^2+\mathbf{Q}^2 - 2\mathbf{PQ}\cos\theta =2(\mathbf{P}^2+\mathbf{Q}^2)$$

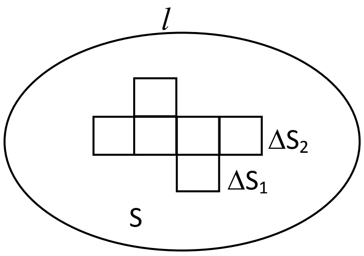
Hence proved.

3. State and prove Stoke's theorem. Write its importance.

Ans. Stoke's theorem : The flux of the curl of a vector function **A** over surface **S** of any shape is equal to the line integral of the vector field **A** over the boundary of surface, i.e.,

$$\iint_s Curl\ \vec{A}.d\vec{S} = \oint_l \vec{A}.d\vec{l}$$

$d\boldsymbol{S}$ represents infinitesimal element of surface area $\boldsymbol{S}$ and $d\boldsymbol{l}$ is an infinitesimal element of the boundary l of that surface.



Proof: Suppose a close curve $\boldsymbol{l}$ encloses vector area $\boldsymbol{S}$ in a vector field $\boldsymbol{A}$. Let the area $\boldsymbol{S}$ be divided into large number of small area $\Delta\boldsymbol{S_1}$, $\Delta\boldsymbol{S_2}$, $\Delta\boldsymbol{S_3}$, …$\Delta\boldsymbol{S_i}$…. etc. having perimeters $\Delta\boldsymbol{l_1}$, $\Delta\boldsymbol{l_2}$, $\Delta\boldsymbol{l_3}$ …$\Delta\boldsymbol{l_i}$ .. etc. respectively. The line integral along the common boundary of the two small area like $\Delta\boldsymbol{S_1}$ and $\Delta\boldsymbol{S_2}$ will cancel each other being in the opposite direction. Hence sum all these line integrals will be equal to the line integral around l the boundary enclosing the whole area $\boldsymbol{S}$ because in traversing the small area all parts of the line integral will cancel out except those parts which are along the outer boundary $\boldsymbol{l}$.

$$\oint_l \vec{A}.d\vec{l} = \sum \oint_{\Delta l_i} \vec{A}.d\vec{l} \qquad (1)$$

$$Curl_n\vec{A} = \lim_{\Delta S_i \to 0} \frac{1}{\Delta S_i} \oint_{\Delta l_i} \vec{A}.d\vec{l} \qquad (2)$$

$$\oint_{\Delta l_i} \vec{A}.d\vec{l} = (Curl\ _n\vec{A})\Delta S_i \qquad (3)$$

$$\sum \oint_{\Delta l_i} \vec{A}.d\vec{l} = \sum (Curl\ _n\vec{A})\Delta S_i \qquad (4)$$

and since $\sum (Curl\ _n\vec{A})\Delta S_i = \iint_s Curl\ \vec{A}.d\vec{S}$ (5)

From Eq. (1), (4) and (5) in

$$\iint_s Curl\ \vec{A}.d\vec{S} = \oint_l \vec{A}.d\vec{l}$$

Hence proved.

Importance:
It help us to convert the line integral of a vector into the surface integral of the curl of that vector and vice-versa.

4. Find the work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve x = t^2+1, y = $2t^2$ from t = 1 to t = 2.

Ans. The work done = $\int_C F.dr = \int_c \left(3xy\hat{i} - 5z\hat{j} + 10x\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$

It is given that particle is moving on xy plane, we assume that z as constant i.e. z=a

as, x = t^2+1 ⇒ dx = 2tdt, y=$2t^2$ ⇒ dy=4tdt, z=a ⇒dz=0

$$\Rightarrow \int_C F.dr = \int_c 3\left(t^2+1\right)2t^2.2tdt - 5a4tdt + 10\left(t^2+1\right)0$$

$$= \int_c \left(12t^5 + 12t^3\right)dt - 20atdt + 0$$

$$= \int_c \left(12t^5 + 12t^3 - 20at\right)dt$$

$$= \int_{t=1}^{t=2} \left(12t^5 + 12t^3 - 20at\right)dt$$

$$= \left[\frac{12t^6}{6} + \frac{12t^4}{4} - \frac{20at^2}{2}\right]_{t=1}^{t=2}$$

= 171-30a

5. An electric field intensity of 0.0686 V/m exists between two points on a conductor. Find the current density J in the conductor. Also calculate the current in conductor if the cross sectional area is 1 mm^2. Given resistivity of the material 1.72 x 10^{-8} Ω -m.

Ans. E = 0.0686 V/m, ρ =1.72 x 10^{-8} Ω -m

σ =1/ρ = 5.81 x 10^7 (Ω -m)$^{-1}$

Current density J = σE = 5.81 x 10^7 x 0.0686

$= 3.99 \times 10^6$ Amp-m^2

Current in the conductor $I = J.A$ (since the cross sectional area is 1 mm^2 = 10^{-6} m^2)

$= 3.99 \times 10^6 \times 1 \times 10^{-6}$

$= 3.99$ A

6. Define the term electric susceptibility. The dielectric constant of Argon at N.T.P. is 1.00538. Calculate the dipole moment induced in each atom of Argon when placed in an electric field of 600 kV/meter.

Ans. **Electric susceptibility**: The ratio of polarization per unit volume P to the net electric field $\varepsilon_0 E$ as modified by the induced charge on the surface of the dielectric is called electric susceptibility (χ_e), i.e.

$$\chi_e = \frac{\boldsymbol{P}}{\varepsilon_0 \boldsymbol{E}}$$

Given dielectric constant of Argon, k=1.00538

Now, $\chi_e = (k - 1) = 1.00538 - 1 = 0.00538$

Since, $P = \chi_e \varepsilon_0 E$

Let n be the number of Argon atom per unit volume, then

$p = P/n = \chi_e \varepsilon_0 E/n$

Number of atom in one gm-atom of a gas at N.T.P

$= 6.023 \times 10^{23}$ atoms in 22.4 liters = 22.4×10^{-3} m^3

$\Rightarrow$ $n = 6.023 \times 10^{23}/22.4 \times 10^{-3} = 2.7 \times 10^{25}$ atom/m^3

$p = \chi_e \varepsilon_0 E/n = 8.85 \times 10^{-12} \times 0.00538 \times 6 \times 10^5 / 2.7 \times 10^{25}$

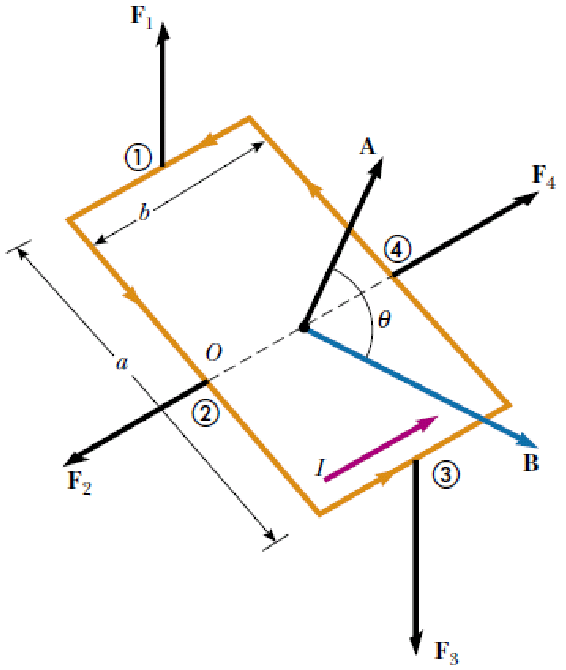
$= 1.058 \times 10^{-33}$ Cm

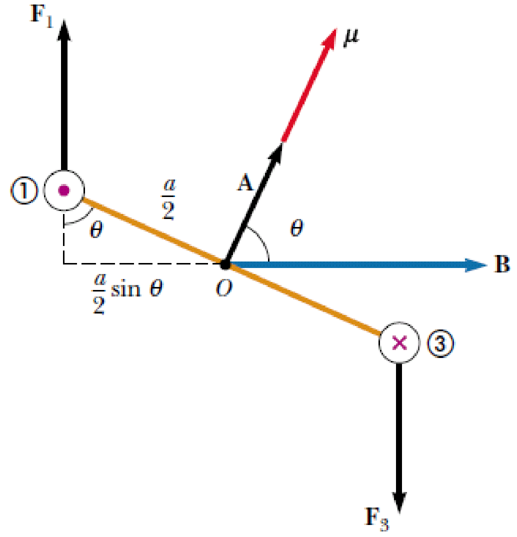
7. What is Lorentz force in magnetostatics? Derive the expression for the torque on a rectangular current loop in magnetic field.

Ans. If a charge q is moving with a velocity $\mathbf{v}$ in the presence of both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ experiences both an electric force $q\mathbf{E}$ and a magnetic force $q\mathbf{v} \times \mathbf{B}$. Then the total force acting on the charge is called the Lorentz force and is given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

Torque on a rectangular current loop in magnetic field





Let us suppose that the uniform magnetic field **B** makes an angle $\theta < 90°$ with a line perpendicular to the plane of the loop, as shown in Figure. For convenience, we assume that B is perpendicular to sides 1 and 3. In this case, the magnetic forces F_2 and F_4 exerted on sides 2 and 4 cancel each other and produce no torque because they pass through a common origin. However, the forces acting on sides 1 and 3, F1 and F3 , form a couple and hence produce a torque about *any point.* The moment arm of F1 about the point O is equal to $(a/2)\sin\theta$. Likewise, the moment arm of F3 about O is also $(a/2)\sin\theta$. Because the net torque about O has the magnitude

$$\begin{aligned}\tau &= F_1\frac{a}{2}\sin\theta + F_3\frac{a}{2}\sin\theta \\ &= IbB\left(\frac{a}{2}\sin\theta\right) + IbB\left(\frac{a}{2}\sin\theta\right) = IabB\sin\theta \\ &= IAB\sin\theta\end{aligned}$$

where A =*ab* is the area of the loop.

8. Obtain the Ampere's law in differential form. Derive the expression for magnetic field due to current in a straight conductor of infinite length.

Ans. According to Ampere's circuital law

$$\oint \boldsymbol{B}\cdot\boldsymbol{dl} = \mu_0\boldsymbol{I}$$

Since $\boldsymbol{I} = \iint_s \boldsymbol{J} \cdot \boldsymbol{ds}$

$$\Rightarrow \oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 \iint_s \boldsymbol{J} \cdot \boldsymbol{ds}$$

According to Stokes theorem

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \iint_s (\boldsymbol{CurlB}) \cdot \boldsymbol{ds} = \iint_s (\nabla \times \boldsymbol{B}) \cdot \boldsymbol{ds}$$

From above equation

$$\iint_s (\nabla \times \boldsymbol{B}) \cdot \boldsymbol{ds} = \mu_0 \iint_s \boldsymbol{J} . \boldsymbol{ds}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

This is Amperes Law in differential form.

Magnetic field due to current in a straight conductor of infinite length

Using Amperes law

Consider a straight wire carrying current I in it. If consider the amperian loop of radius *a* around the wire and apply the amperes law, i.e.

$$\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 \boldsymbol{I}$$

Since B is constant around the wire at a distance *a*

$$\Rightarrow \boldsymbol{B} \oint \boldsymbol{dl} = \mu_0 \boldsymbol{I}$$

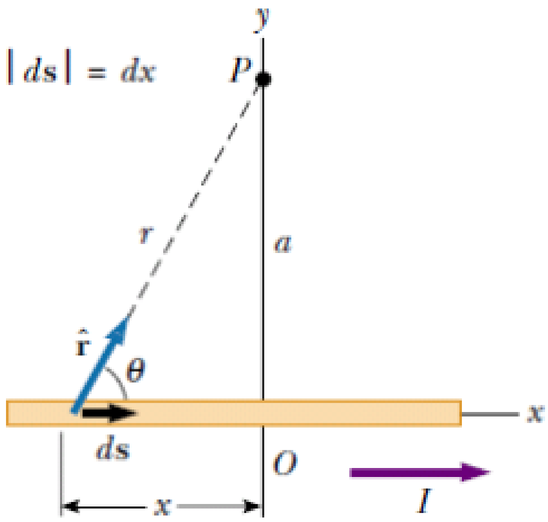
$$\Rightarrow \boldsymbol{B.2\pi a = \mu_o I}$$

$$\Rightarrow \boldsymbol{B} = \frac{\mu_0 \boldsymbol{I}}{2\pi \boldsymbol{a}}$$

Or

Using Biot-Savart Law

Consider a straight wire carrying a constant current *I* and placed along the *x* axis as shown in Figure. Consider a length element *ds* located at a distance *r* from P. The direction of the magnetic field at point P due to the current in this element is out of the page. Since all of the current elements *I ds* lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Thus, we have the direction of the magnetic field at point P, and we need only find the magnitude.



According to the Biot–Savart law, the magnetic field at point P due to element *ds* is given by

$$dB = \frac{\mu_0 I}{4\pi}\frac{dx\, sin\theta}{r^2}$$

The total magnetic field at point P due to all elements, subtending angles ranging from θ_1 to θ_2 is given by

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\frac{dx\, sin\theta}{r^2}$$

Since $r = a / sin\ \theta \Rightarrow a\ cosec\ \theta$

and $x = a\ cot\theta \Rightarrow dx = -\ a\ cosec^2\theta d\theta$

putting the value of x and r we get

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2} sin\theta d\theta \quad = \quad \frac{\mu_0 I}{4\pi a}\left(cos\theta_1 - cos\theta_2\right)$$

For infinitely long conductor $\theta_1 = 0$ and $\theta_2 = \pi$

$$B = \frac{\mu_0 I}{2\pi a}$$